Comment on "Gravity Gradient Torque for an Arbitrary Potential Function"

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THE purpose of this comment on Ref. 1 is to show that it is unnecessary to solve an eigenvalue problem to obtain an explicit expression for the gravity gradient torque caused by an arbitrary potential function. If we let G represent the gravity gradient tensor after transformation into C_s (spacecraft coordinates) rather than in C_a (attracting body coordinates) as in Ref. 1, we may replace Eq. (11) of Ref. 1 by Eq. (11a).

$$G = \sum_{i=1}^{3} \sum_{j=1}^{3} G_{ij} \psi_i \psi_j^T \tag{11a}$$

where the ψ_i are simply the spacececraft coordinate system unit vectors.

$$\psi_1 = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}, \quad \psi_2 = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}, \quad \psi_3 = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \tag{1b}$$

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We then follow through the same development as Ref. 1, Eqs. (12-17) with the final result:

$$\tau_{gg} = \sum_{i=1}^{3} \sum_{j=1}^{3} G_{ij} \tilde{\psi}_i J \psi_j \tag{17a}$$

We now expand Eq. (17a) in components using the ψ_i from Eq. (1b). The symmetry of both tensors permits a slight simplification.

$$\tau_{gg} = \begin{Bmatrix} G_{12}J_{31} - G_{31}J_{12} + (G_{22} - G_{33})J_{23} - G_{23}(J_{22} - J_{33}) \\ G_{23}J_{12} - G_{12}J_{23} + (G_{33} - G_{11})J_{31} - G_{31}(J_{33} - J_{11}) \\ G_{31}J_{23} - G_{23}J_{31} + (G_{11} - G_{22})J_{12} - G_{12}(J_{11} - J_{22}) \end{Bmatrix}$$

A more elegant but less computationally efficient expression of Eq. (2b) is Eq. (3b).

$$\tau_{gg_n} = \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \delta_{nij} G_{ik} J_{jk} \tag{3b}$$

Here δ_{ijk} is the Levi-Civita tensor density, which equals $+1$ when i, j, k are even perturbations of 1, 2, 3; equals -1 when i, j, k are odd perturbations of 1, 2, 3; and equals zero when i, j, k are not all different. Equation (3b) makes obvious the covariance of the result. That is, Eq. (2b) or (3b) is valid in any coordinate system, as long as τ_{gg}, G, and J have their components expressed in the same coordinate system.

References

[1]Glandorf, D.R., "Gravity Gradient Torque for an Arbitrary Potential Function," *Journal of Guidance, Control, and Dynamics,* Vol. 9, Jan.-Feb. 1986, pp. 122-124.

Book Announcements

YOUNG, P., University of Lancaster, ***Recursive Estimation and Time-Series Analysis,*** Springer-Verlag, New York, 1984, 300 pages. $31.50.

Purpose: This text is primarily intended as an introduction to recursive estimation for undergraduate and first-year graduate students. It is also considered as the theoretical basis for the CAPTAIN computer package (Computer Aided Program for Time-series Analysis and the Indentification of Noise Systems).

Contents: Introduction. Recursive estimation: a tutorial introduction. Recursive estimation and stochastic approximation. Recursive least squares regression analysis. Recursive estimation of time-variable parameters in regressive models. The time-series estimation problem. The instrucmental variable (IV) method of time-series analysis. Optimal instrumental variable methods of time-series model estimation. Alternative recursive approaches to time-series analysis. Recursive estimation: a general tool in data analysis and stochastic model building. Epilogue. Appendices. Indices.

WONHAM, W.M., University of Toronto, ***Linear Multivariable Control,*** third edition, Springer-Verlag, New York, 1985, 334 pages. $46.20.

Purpose: The goal of this text is to present a geometric approach to the structural synthesis of multivariable control systems that are linear, time invariant and of finite dynamic order. The text is addressed to graduate students specializing in control, to engineering scientists involved in control systems research and development, and to mathematicians interested in systems control theory.

Contents: Mathematical preliminaries. Introduction to controllability. Controllability, feedback, and pole assignment. Observability and dynamic observers. Disturbance decoupling. Controllability subspaces. Tracking and regulation I: output regulation. Tracking and regulation II: output regulation with internal stability. Tracking and regulation III: structurally stable synthesis. Noninteracting control I: basic principles. Noninteracting control II: efficient compensation. Noninteracting control III: generic solvability. Quadratic optimization I: existence and uniqueness. Quadratic optimization II: dynamic response. Index.